ES 205 Analysis and Design of Engineering Systems

Lesson 18

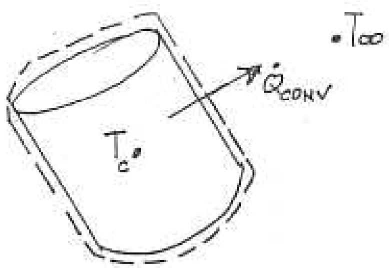
Thermal Systems Board Problems

Example 1

A small copper casting is removed from a mold at 650°C and cooled in a large tank of fluid (h_c = 790 W/m^2-K, T_∞= 75°C). Assume the fluid temperature is constant. The properties of copper are: ρ = 8933 kg/m^3, C_v = 383 J/kg-K, and k = 399 W/m-K. The volume of the connector is 1.75 cm^3 and its surface area is 3.5 cm^2.

Determine the system time constant, the steady-state temperature of the casting, and the amount of time that the casting must remain in the fluid to cool from its initial 650°C to 100°C.

SYSTEM : CASTING



NO HEAT TRANSFER INTO THE SYSTEM.
NO $\dot{m}$-TERMS
NO $\dot{W}$ TERMS
ASSUME RADIATION NEGLIGIBLE
TENTATIVELY ASSUME LUMPED CAPACITANCE

COE $$\frac{d}{dt}(U) = \cancel{\dot{Q}_{IN}}^{\,0} - \dot{Q}_{OUT} + \cancel{\dot{W}_{IN}}^{\,0} - \cancel{\dot{W}_{OUT}}^{\,0}$$

$$\frac{d}{dt}(mcT_c) = -hA(T_c - T_\infty)$$

$$\rho V c \dot{T}_c = -hA(T_c - T_\infty) \quad \blacktriangleleft \text{EOM}$$

BIOT NO: $$B_i = \frac{h V}{k A_s}$$

$$= \frac{\left(790 \frac{W}{m^2 K}\right)(3.5\ cm^3)}{\left(399 \frac{W}{m K}\right)(1.75\ cm^2)}\left(\frac{m}{100\ cm}\right)$$

$$= 0.040$$

Since $Bi < 0.10$, lumped-capacitance assumption is valid and temp. T_c of casting can be modeled as uniform.

$$\rho \forall c \dot{T}_c = -hA(T_c - T_\infty)$$

$$\frac{\rho \forall c}{hA} \dot{T}_c + T_c = T_\infty \qquad \text{STD. FORM}$$

$$\tau \dot{T}_c + T_c = T_\infty \qquad \text{WHERE} \quad \tau = \frac{\rho \forall c}{hA}$$

STEADY STATE, $\dot{T} = 0$

$$\cancel{\tau \dot{T}_c} + T_c = T_\infty$$

$$T_{c\,ss} = T_\infty$$

$$= 75°C$$

$$\tau = \frac{\rho \forall c}{hA} = \frac{(8933 \frac{kg}{m^3})(1.75\ cm^3)(383 \frac{J}{kg\,K})}{(790 \frac{W}{m^2 K})(3.5\ cm^2)} \left(\frac{m}{100\ cm}\right)$$

$$= 21.6 \text{ sec}$$

ODE: $21.6\, \dot{T}_c + T_c = 75°C$

IC: $T_c(0) = 650°C$

SOLUTION: 1^{st} ORDER, STEP RESPONSE

$$T_c(t) = (T_{co} - T_{ss})\, e^{-t/\tau} + T_{css}$$

$$\underline{T_c(t) = (650 - 75) e^{-t/21.6} + 75 \quad °C}$$

TIME TO REACH 100°C:

$$100 = (650 - 75) e^{-\frac{t}{21.6}} + 75$$

SOLVE FOR t

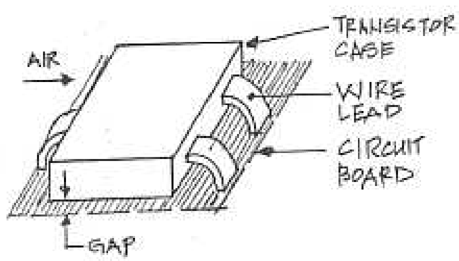
$$0.043 = e^{-t/21.6}$$

$$\underline{t = 68\ s}$$

Example 2

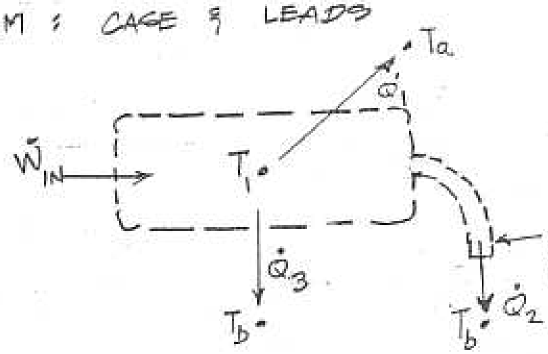
A transistor case is mounted on a circuit board whose surface temperature is maintained at 35°C. Air at 20°C flows over the upper surface (4mm by 8mm) of the case with a convection coefficient of 50 W/m^2-K. Three wire leads (each of cross-section 1 mm by 0.25 mm) conduct heat from the case to the circuit board. The gap between the case and the board is 0.2 mm and is filled with stagnant air. Assume the case is isothermal and neglect radiation. The thermal conductivity of the wire leads is 25 W/m-K; the thermal conductivity of air is 0.0263 W/m-K.

Determine the steady-state case temperature when 150 mW are generated by the transistor.



SYSTEM : CASE & LEADS

T_a : AIR
T_1 : CASE
T_b : BOARD



$\dot{Q}_1$: CONVECTION ACROSS TOP SURFACE
$\dot{Q}_2$: CONDUCTION THRU LEADS
$\dot{Q}_3$: CONDUCTION THRU STAGNANT AIR GAP.

NO MASS-FLOW TERMS

CONS. OF ENERGY

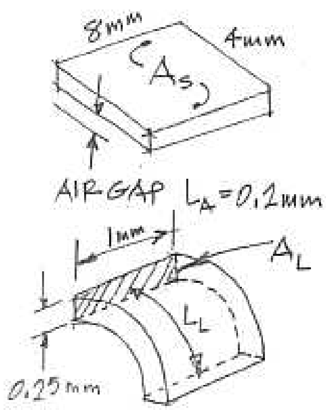
$$\underbrace{\frac{d}{dt}U}_{SS} = \dot{W}_{IN} - \dot{Q}_1 - 3\dot{Q}_2 - \dot{Q}_3$$

$$\dot{W}_{IN} = \dot{Q}_1 + 3\dot{Q}_2 + \dot{Q}_3$$

$$\dot{W}_{IN} = hA_S(T_1 - T_a) + 3\frac{k_L A_L}{L_L}(T_1 - T_b) + \frac{k_A A_S}{L_A}(T_1 - T_b) \quad (1)$$

STEADY-STATE, SO $\frac{dU}{dt} = 0$, NO NEED TO CHECK Bi

LENGTHS & AREAS



$A_S = 3.2 \times 10^{-5}$ m^2

$L_A = 0.0002$ m

$A_L = 2.5 \times 10^{-7}$ m^2

$L_L = 0.004$ m

EQN: (1)

UNKN: T_1

PARAMETERS

h L_L A_L
k_L L_A
k_A A_S

INPUTS

$T_a = 20°C$

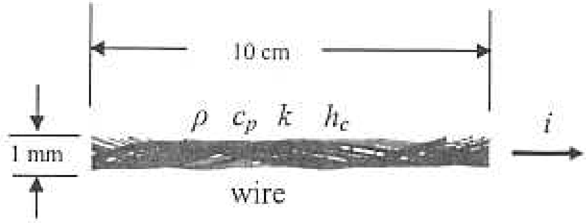
$T_b = 35°C$

$\dot{W}_{IN} = 0.15$ W

SOLVE $\longrightarrow$ $T_1 = 47°C$

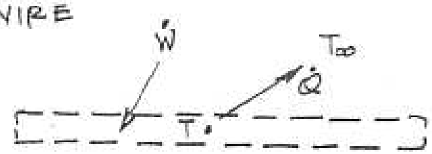
Example 3

You heat a piece of aluminum wire by passing an electrical current through it. The wire has a diameter of 1 mm and a length of 10 cm, giving it an electrical resistance of 0.2 Ω. The initial temperature of the wire is 25°C and the aluminum is surrounded by air at 25°C and $h_c = 20$ W/m^2-K. (Aluminum properties: $\rho = 2787$ kg/m^3, $C_v = 833$ J/kg-K, and $k = 164$ W/m-K.)



For a steady 1.0 Amp current in the wire, determine the system time constant, the steady-state temperature of the wire, and the temperature of the wire 1 minute after the steady current begins.

SYSTEM : WIRE



$\dot{W}$ DUE TO ELEC. CURRENT

$\dot{Q}$ DUE TO CONVECTION

TENTATIVELY ASSUME WIRE TEMP. T IS UNIFORM.

$$\frac{dU}{dt} = \dot{W} - \dot{Q}$$

$$\frac{d}{dt}(\rho \forall c T) = \dot{W} - h A_s (T - T_\infty)$$

FOR CONSTANT ρ AND c

$$\rho \forall c \dot{T} = \dot{W} - h A_s (T - T_\infty)$$

$$Bi = \frac{h \forall}{k A_s} \qquad \text{WHERE } \forall = \frac{\pi d^2}{4} L = \frac{\pi (0.001\,m)^2}{4}(0.1\,m)$$

$$= 7.85 \times 10^{-8}\ m^3$$

$$\text{AND } A_s = \pi d L = \pi (0.001\,m)(0.1\,m)$$

$$= 3.14 \times 10^{-4}\ m^2$$

$$Bi = \frac{20 \frac{W}{m^2 K}}{164 \frac{W}{m K}} \, \frac{7.85 \times 10^{-8}\ m^3}{3.14 \times 10^{-4}\ m^2} = 3 \times 10^{-5} < 0.1$$

∴ LUMPED CAPACITANCE O.K.

$$\frac{\rho \forall c}{h A_s}\dot{T} + T = T_\infty + \frac{\dot{W}}{h A_s}$$

$$\tau = \frac{\rho \forall c}{h A_s}$$

AT S.S., $\dot{T} = 0 \Rightarrow T_{ss} = T_\infty + \frac{\dot{W}}{h A_s}$

$\dot{W} = i^2 R = (1.0\,A)^2(0.2\,\Omega) = 0.2\,W$

$$T_{ss} = 25°C + \frac{0.2\,W}{20\frac{W}{m^2K}(3.14\times10^{-4})m^2}$$

$$T_{ss} = 57°C$$

$$\tau = \frac{\rho \forall c}{h A_s} = \frac{\left(2787\,\frac{kg}{m^3}\right)(7.85\times10^{-8}\,m^3)\left(833\,\frac{J}{kg\,K}\right)}{\left(20\,\frac{W}{m^2K}\right)(3.14\times10^{-4}\,m^2)}$$

$$= 29\text{ sec}$$

$$29\dot{T} + T = 57\ °C \qquad T(0) = 25°C$$

SOLUTION

$$T(t) = (T_0 - T_{ss})e^{-t/\tau} + T_{ss}$$
$$= (25-57)e^{-t/29} + 57$$

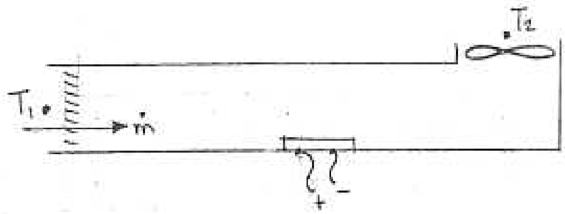
$$T(t) = 57 - 32e^{-t/29}$$

After 60s, $T(60) = 57 - 32e^{-60/29}$

$$T = 53°C$$

Example 4

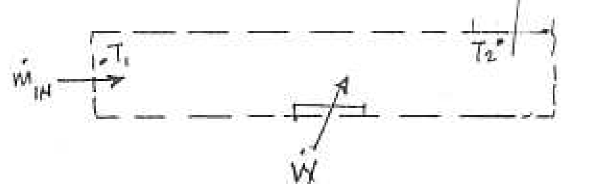
A computer has a problem with its main processor overheating. To solve the problem it is proposed to install a fan as shown below.



Assume all surfaces are perfectly insulated and that air is an ideal gas entering at a constant temperature T_1. Determine:

- An expression for the steady-state temperature of the air T2 when the chip is energized.
- An expression for the steady-state temperature of the chip when it is energized.
- The differential equations that model the temperature of the air and the temperature of the chip.

SYSTEM : AIR PLUS CHIP



NO HEAT TRANSFER

$\dot{W}$ DUE TO ENERGIZING THE CHIP.

$\dot{m}$ IS THE FLOW OF AIR

COE :

$$\underbrace{\frac{dU}{dt}}_{SS} = \underbrace{\dot{Q}_{IN}}_{0} - \underbrace{\dot{Q}_{OUT}}_{0} + \dot{W}_{IN} - \underbrace{\dot{W}_{OUT}}_{0} + \dot{m}(h_1 - h_2)$$

$$0 = \dot{W}_{IN} + \dot{m}(h_1 - h_2)$$

SS → No BIOT.

$$\dot{W}_{IN} = \dot{m}(h_2 - h_1)$$

$$= \dot{m} c_p (T_2 - T_1)$$

$$T_2 = T_1 + \frac{\dot{W}_{IN}}{\dot{m} c_p}$$

SYSTEM : CHIP ONLY

T : chip temp.
T_a : air temp near chip
$\dot{Q}$: convective

COE : $\frac{dU}{dt}_{sys} = \dot{W} - \dot{Q}_{OUT}$

SS: $\frac{dU}{dt} = 0$

$$0 = \dot{W} - hA(T - T_a)$$

$$T = T_a + \frac{\dot{W}}{hA}$$

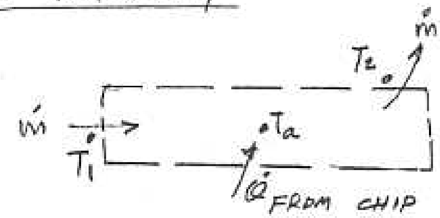
h: conv. coeff. (NOT enthalpy)

WE DON'T KNOW T_a, WE ONLY KNOW T_1 & T_2

LET'S ESTIMATE $T_a = \frac{1}{2}(T_1 + T_2)$

$$T = \frac{1}{2}(T_1 + T_2) + \frac{\dot{W}}{hA}$$

SYSTEM : AIR ONLY



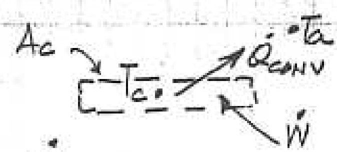
SUBST $\underbrace{T_2 = 2T_a - T_1}$

COE (AIR) $\frac{dU}{dt}_{sys} = \dot{Q} + \dot{m}(h_1 - h_2)$

$$m_a c_{pa} \dot{T}_a = hA_c(T_c - T_a) + \dot{m} c_{pa}(T_1 - T_2) \quad (1)$$

EQN	UNKN.
(1)	T_a
(2)	T_c

SYSTEM : CHIP ONLY



COE (CHIP) $\frac{dU}{dt}_{sys} = \dot{W} - \dot{Q}$

$$m_c c_{vc} \dot{T}_c = \dot{W} - hA_c(T_c - T_a) \quad (2)$$

INPUTS
T_1
$\dot{W}_{IN}$

PARAM.

m_a : air mass
m_c : chip mass
c_{pa} : air c_p
c_{vc} : chip c_v
A_c : chip area
$\dot{m}$: mass-flow-rate of air
h : conv. coeff.